A Novel Approach for Simulation of Fixed Bed Regenerator

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Abstract—Fixed-bed regenerators are compact heat exchangers in which heat is absorbed from the high temperature flue gases and released to the low temperature inlet gas using high heat capacity material matrix. The heat transfer in the fixed bed heat exchangers is govern by the coupled partial differential equations(PDEs). The computational time for solving these coupled PDEs is very large using the numerical methods. In this paper these equations are solved using the finite-difference method and it is observed that the computational time of the solution got reduce without any significant variation in the results by using the wavelets. The wavelets are the functions having some value for a short duration and zero otherwise. It helps in analyzing the rapidly changing transient signals. The grid size changes according to the nature of the curve. In the present work two wavelets *i.e.*haar wavelet and daubechies wavelets are used for the grid adaption and the result obtained by them is compared.

Index Terms—Coupled partial-differential equation,daubechies wavelet,fixed-bed regenerators, grid-adaption, haar wavelet, rapidly changing transient signals, wavelets.

1 INTRODUCTION

The partial differential equations encountered in heat and mass transfer problems, involving a moving temperature profile or boundary, are complicated to solve analytically. These equations are usually coupled transient parabolic partial differential equations. Solution of these equations using numerical methods takes large amount of computational time. The problem is to find an efficient numerical approximation method to solve these coupled transient PDEs. The method should be able to analyze the temperature front moving with time and maintain an effective grid discretization of the spatial variable.

These type of coupled transient equations are occurred in many situations. One of them is fixed-bed regenerator. Fixed-bed heat exchangers are used in industries or power plants to recover heat from hot exhaust gases and then reuse this heat to preheat incoming surrounding air. This increases efficiency of the plant and helps in controlling the environmental emissions. A stream of high temperature flue gases is made to flow over the regenerator. The regenerator absorbs heat from the flue gases and stores it in the matrix made of any high heat capacity solid material so the temperature of flue gasses emitting to the surroundings gets reduce and the temperature of the solid material matrix increases. After a certain duration the flow of the hot gas is stopped. This is called charging period of the regenerator. Now a stream of cold inlet gas usually surrounding air is made to flow over the heated or charged regenerator matrix for a certain period. The cold stream receives heat from the charged matrix and gets preheated before entering to the plant. This cools down the heated regenerator matrix and brings its temperature down to the initial temperature. This is called the discharging period of the regenerator. The preheating insures that the lesser energy is required to heat this inlet air inside the plant which results in an increment in the efficiency of the plant. At least two beds are required for the continuous operation of the fixed-bed regenerator.

The concentration and temperature profiles of a fixed-bed regenerators shows dynamic fronts. The methodsfor solving these type of problems are inefficient mainly because oftaking uniform dense grids alongcomplete bed length for all time levels. This computational time can be reduced by the methodswhich are based on non-uniform grids and can adapt the changes in the solution dynamically. This requirement can be achieved by the use of wavelets. The concept of wavelet was introduced in applied mathematics and physics by the end of the 1980s by Daubechies and Mallat.The wavelets functions break down the data into different frequency components, and then study each component with a resolution matched to its scale.A set of wavelet coefficients is generated for different resolutions and spatial location. These coefficients are compared with a threshold value. The points having the coefficient value lesser than the threshold value are eliminated from the grids. This eliminates the problem of dense grids for complete domain.

2 WAVELETS THEORY

Wavelets arefunctions which are non-zero for very short duration and having a zero integrated value. All set of functions of a wavelet family generates from the single wavelet function $\psi(x)$called mother wavelet by scaling and translation operation.

$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k) \qquad j,k \in Z$$

Different scaled and translated versions of the wavelet function can be obtained by varying the values for j and k respectively. Wavelets are derived from a scaling function$\phi(x)$.

$$\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k) \qquad j,k \in Z$$

Wavelet transform decomposes a discrete signal into two sub-signals of half of its length. The two sub-signals relate to two type of wavelet coefficients *i.e.* approximation coefficients and detailed coefficient. If a function is approximately constant over a time period or a spatial region, its detailed wavelets coefficient becomes approximately zero for that region.

2.1 Haar Wavelet

Haar wavelet is the simplest and the oldest among all wavelets and provide foundation for understanding all other wavelets.

Haar scaling function is defined as

$$\phi(x) = \begin{cases} 1, & for\ 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

Haarwaveletfunction is defined as

$$\psi(x) = \begin{cases} 1, & for\ 0 \leq x \leq 1/2 \\ -1, & for\ 1/2 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

2.2 Daubechies(db4) wavelet

Daubechies wavelets extends the haar wavelets by using longer filters, that produce smoother scaling functions and wavelets. The difference between the Haar transform and the daubechies transform lies in the definition of scaling signals and wavelets. The db4 scaling signals has the support of four time or space units. The db4 scaling coefficients are defined as

$$\alpha_1 = \frac{(1 + \sqrt{3})}{4\sqrt{2}} \qquad \alpha_2 = \frac{(3 + \sqrt{3})}{4\sqrt{2}}$$

$$\alpha_3 = \frac{3 - \sqrt{3}}{4\sqrt{2}} \qquad \alpha_4 = \frac{(1 - \sqrt{3})}{4\sqrt{2}}$$

Similarly, db4 wavelet numbers are defined as

$$\beta_1 = \frac{(1 - \sqrt{3})}{4\sqrt{2}} \qquad \beta_2 = \frac{(\sqrt{3} - 3)}{4\sqrt{2}}$$

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$$\beta_3 = \frac{3 + \sqrt{3}}{4\sqrt{2}} \qquad \beta_4 = \frac{(-1 - \sqrt{3})}{4\sqrt{2}}$$

These scaling coefficients and wavelet numbers are used to construct the 1st level db4 scaling and wavelet functions.

3 MATHEMATICAL MODEL

The fig. 1 shows the model of heat transfer in fixed-bed heat regenerator.
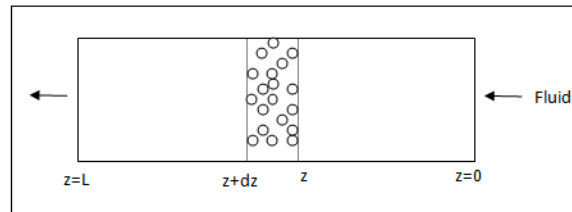

Fig. 1. Heat transfer model of a bed

Following assumptions are made to formulate the mathematical model.

1. Thermal properties of the gas and solid are constant.
2. The velocity and temperature fields of each fluid at the inlet are uniform over the flow cross section and do not change with time.
3. All balls in the bed are identical and have a similar size.
4. There is no heat loss from the regenerator's wall.
5. There is no internal heat generation in the regenerator.
6. Balls have a single contact point which results negligible axial conduction in the solids.
7. The mass flow rates of hot and cold streams are constant.

The heat balance forthe differential element shown in fig. 1 leads to following governing equations for solid and gaseous phase.

For gaseous phase

$$k_e \left(\frac{\partial^2 T_g}{\partial z^2} \right) - C_{pg}\rho_g v_g \left(\frac{\partial T_g}{\partial z} \right) + h_p a_s (T_s - T_g)/\varepsilon - \frac{4U}{D_b \varepsilon} (T_g - T_a) = C_{pg}\rho_g \partial T_g \partial t \tag{1}$$

For solid phase

$$C_{ps}\rho_s (1 - \varepsilon) \left(\frac{\partial T_s}{\partial t} \right) = h_p a_s (T_a - T_s) \tag{2}$$

In order to find the temperature variation along the bed length, these equations need to be solved simultaneously along with the following initial and boundary condition.

The initial condition for the problem is given by

$k_s(Wm^{-1}K^{-1})$	1.9
$U(Wm^{-2}K^{-1})$	0

The boundary conditions are given by

$$k_e \left(\frac{\partial T_g}{\partial z}\right) = C_{pg}\rho_g v_g \varepsilon (T_g - T_{gi}) \quad for\ t > 0\ and\ z = 0$$

$$\left(\frac{\partial T_g}{\partial z}\right) = 0 \quad for\ t > 0\ and\ z = L$$

4 MATERIALS AND METHOD

These equations are solved by finite-difference method using implicit scheme. Central difference approach is taken to approximate the differential grids.

A program is made in MATLABto solve these equations. The physical parameters used to solve the equations are shown in Table 1.

TABLE 1: DIFFERENT PARAMETERS TAKEN FOR THE FIXED-BED REGENERATOR

Packing material	Gravels
Bed length(m)	2
Packing density (kgm^{-3})	2200
Gas density(kgm^{-3})	0.935
Specific heat solid(Jkg^{-1}K^{-1})	840
Specific heat gas(Jkg^{-1}K^{-1})	1050
Bed void	0.41
v_g(ms^{-1})	0.08
h_p(Wm^{-2}K^{-1})	61

5 RESULTS AND DISCUSSION

Following results are obtained by solving these partial differential equations simultaneously using finite-difference method for equal grids. The inlet temperature of the gas is taken as 400^{0}C.

Fig. 2 shows the variation of temperature along bed length obtained during charging period for the inlet velocity 0.08 ms^{-1} at different time levels *i.e.* 60, 120, 180, 240 and 300 min using finite-difference method for equal 1024 grids for all time levels. The computational time of the program for this case is obtained as 847 s.
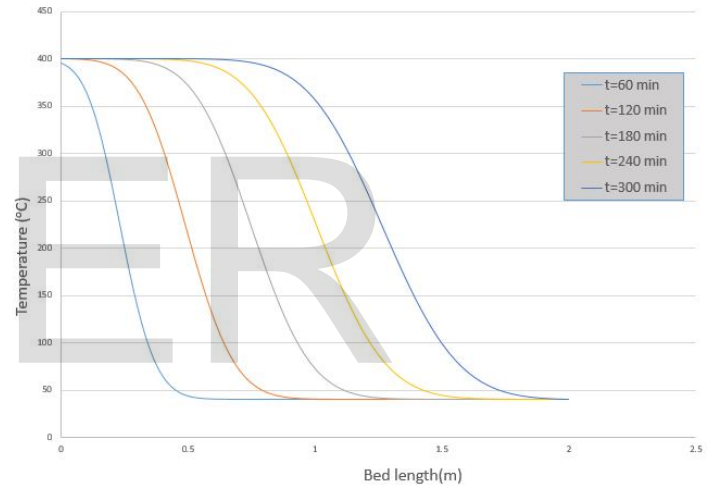

Fig. 2. Variation of temperature along bed length obtained at different time levels using FDM for equal grids

Fig. 3 shows the results by using haar wavelet for the inlet velocity 0.08 ms^{-1}. The computational time of the program for this case is obtained as 314 s.
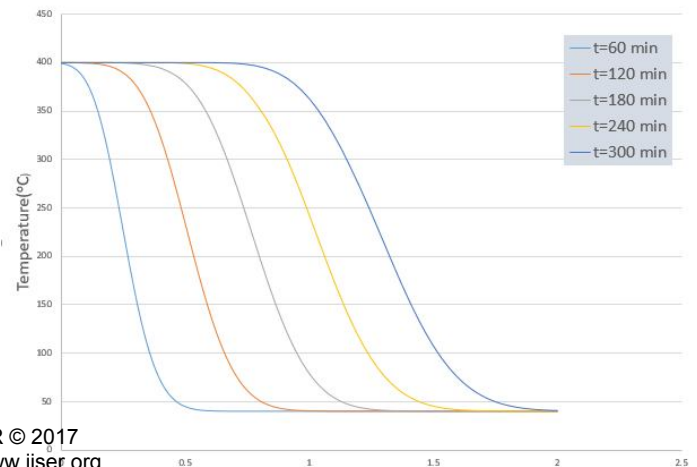
Fig. 3. Variation of temperature along bed length obtained at different time levels using Haar wavelet

Fig. 4 shows the results by using db4 wavelet for the inlet velocity 0.08 ms^{-1}. The computational time of the program for this case is obtained as 173 s.
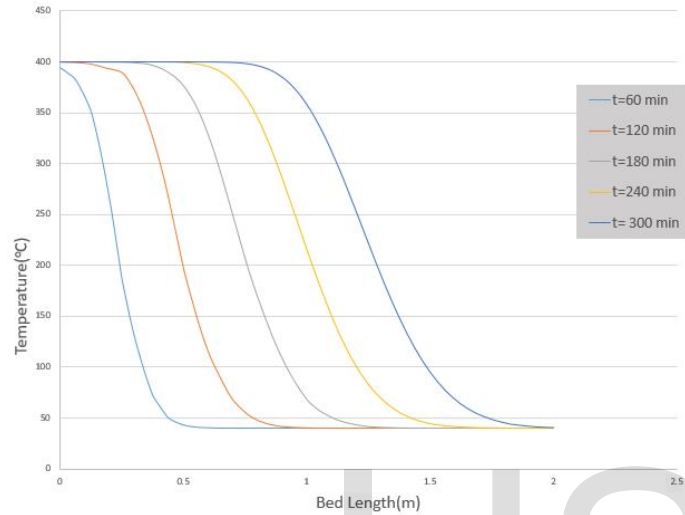

Fig. 4. Variation of temperature along bed length obtained at different time levels using db4 wavelet

Fig. 5 and 6 shows the distribution of grid points for different resolution levels at 60 min for haar and daubechies wavelets respectively.
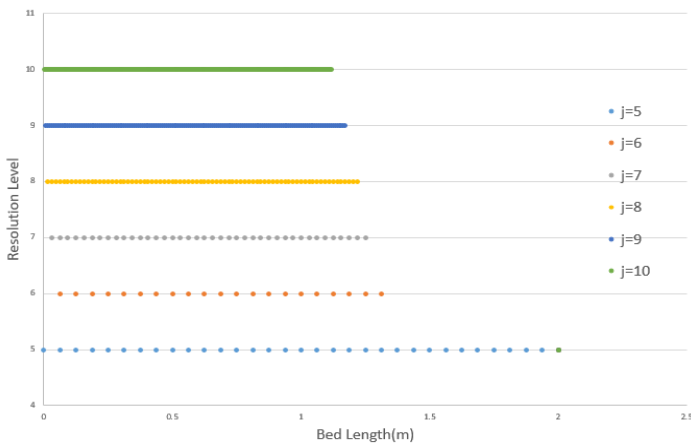

Fig. 5. Grid pattern for haar wavelet at t=60 min

Fig. 6. Grid pattern for db4 wavelet at t=60 min

From the grid pattern for haar and db4 wavelet in can be observed that the high resolution grids are only at the beginning of the bed where the temperature front is presentat t=60 min.

6 CONCLUSIONS

This study shows the advantage of wavelet based adaptive methods over the simple numerical methods. The results show that the computational time in case of wavelet based method was lesser than the simple finite-difference method for same results due to reduction in grid density where curve is smooth and increment in grid density when there are steep changes.The computation becomes 62.9% faster in case of haar wavelet and 79.5% faster in case of db4 wavelet.The computational time using db4 wavelet is minimum.This may be due to the use of longer wavelet filter in case of db4 wavelets that has support of four space units.

NOMENCLATURE

a_s	Specific area of packing solids in bed
C_{pg}	Specific heat of gas stream
C_{ps}	Specific heat of packing solids
D_b	Diameter of bed
h_p	Gas-solid heat-transfer coefficient
k_e	Effective axial thermal conductivity in packed bed
T_g	Gas temperature
T_{gi}	Temperature of influent gas stream
T_s	Solids temperature
T_{si}	Initial solid temperature
t	Time
v_g	Superficial gas velocity in packed bed
ε	Void fraction of packed bed
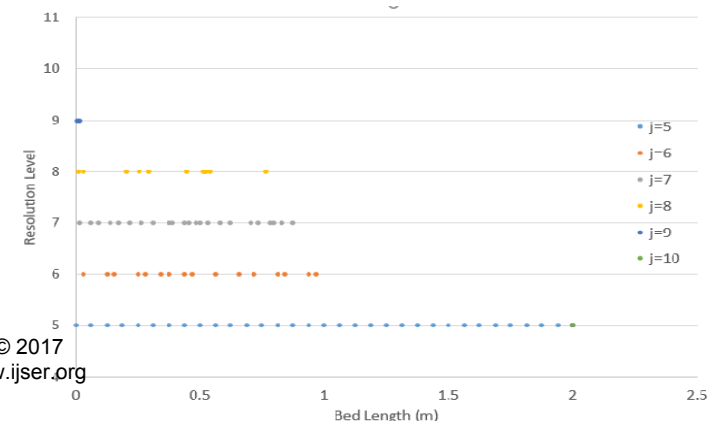
ρ_g Density of gas stream

ρ_s Density of packing solids

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